Limits of Learning in Incomplete Networks

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Background: Incomplete Networks

- Network data is often incomplete
- Acquiring more data is often expensive and/or hard
- Research question: Given a networked dataset and limited resources to collect more data, how can you get the most bang for your buck?

Two general approaches to network completion

Don’t collect more data

Collect more data

Two general approaches to network completion

Don’t collect more data

Assume a network model

Combine network model with incomplete data to get a model of the network structure

Infer missing data from this model

- [Kim et al. 2011]
- [Chen et al. 2018]

Collect more data

Two general approaches to network completion

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Collect more data

Estimate Statistics from partially observed network

- [Soundarajan et al. 2015]
- [Soundarajan et al. 2016]

Utilize an explore-exploit approach

- [Pfeiffer III et al. 2014]
- [Soundarajan et al. 2017]
- [Murai et al. 2018]
- [Madhawa et al. 2018]
- This work!

Our solution: Network Online Learning (NOL)

- Research Question: Given a networked dataset and limited resources to collect more data, how can you get the most bang for your buck?

- Learn to grow an incomplete network through sequential, optimal queries (to some API)
- Agnostic to both *data distributions* and *sampling method*
- Interpretable features that are computable online

Assumptions

We assume...

We know the API access model (complete vs incomplete queries).

The underlying network is static (probing the same node twice gives no new information).

We do not assume...

A model of the underlying graph.

How the initial sample was collected.

Network Online Learning (NOL)

Inputs:

- $\hat{G}_0$: Incomplete network
- *b* : probing budget
- *r* : reward function

Probe $\hat{G}_0$
b times to maximize cumulative reward

Output:

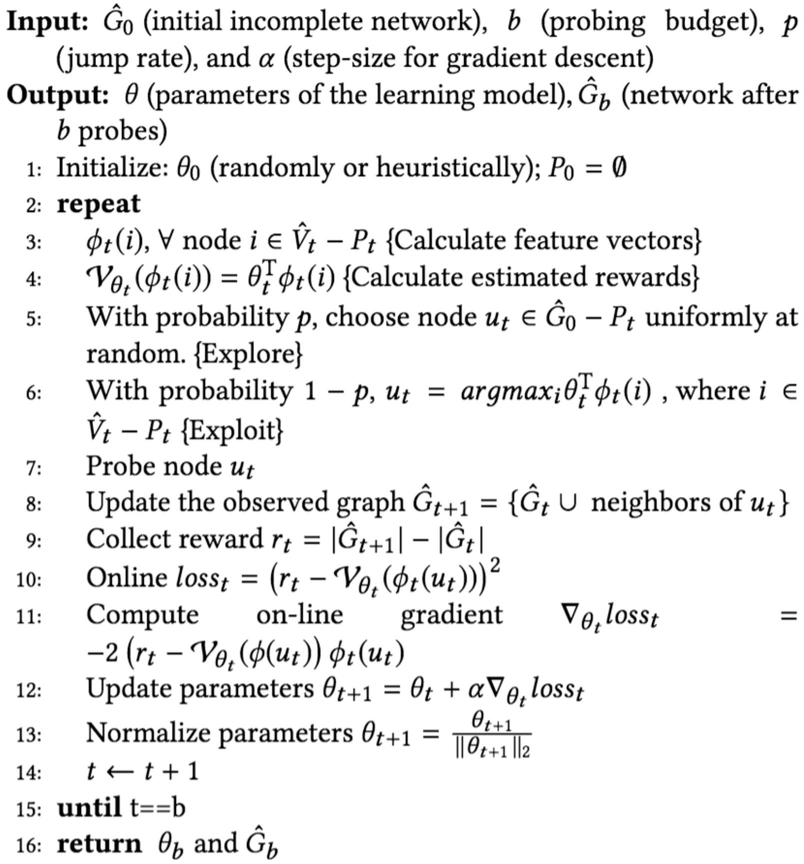
Network after *b* probes

$$\hat{G}_b \approx G$$

Example reward function:

- *number of new nodes observed*

NOL algorithm



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Input: $\hat{G}_0$ (initial incomplete network), $b$ (probing budget), $p$ (jump rate), and $\alpha$ (step-size for gradient descent)
Output: $\theta$ (parameters of the learning model), $\hat{G}_b$ (network after $b$ probes)
Initialize: $\theta_0$ (randomly or heuristically); $P_0 = \emptyset$
repeat
    $\phi_t(i), \forall$ node $i \in \hat{V}_t - P_t$ {Calculate feature vectors}
    $\mathcal{V}_{\theta_t}(\phi_t(i)) = \theta_t^{\mathrm{T}} \phi_t(i)$ {Calculate estimated rewards}
    With probability $p$, choose node $u_t \in \hat{G}_0 - P_t$ uniformly at random. {Explore}
    With probability $1 - p$, $u_t = argmax_i \theta_t^{\mathrm{T}} \phi_t(i)$ , where $i \in \hat{V}_t - P_t$ {Exploit}
    Probe node $u_t$
    Update the observed graph $\hat{G}_{t+1} = \{\hat{G}_t \cup$ neighbors of $u_t\}$
    Collect reward $r_t = |\hat{G}_{t+1}| - |\hat{G}_t|$
    Online $loss_t = \left(r_t - \mathcal{V}_{\theta_t}(\phi_t(u_t))\right)^2$
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until t==b
return $\theta_b$ and $\hat{G}_b$
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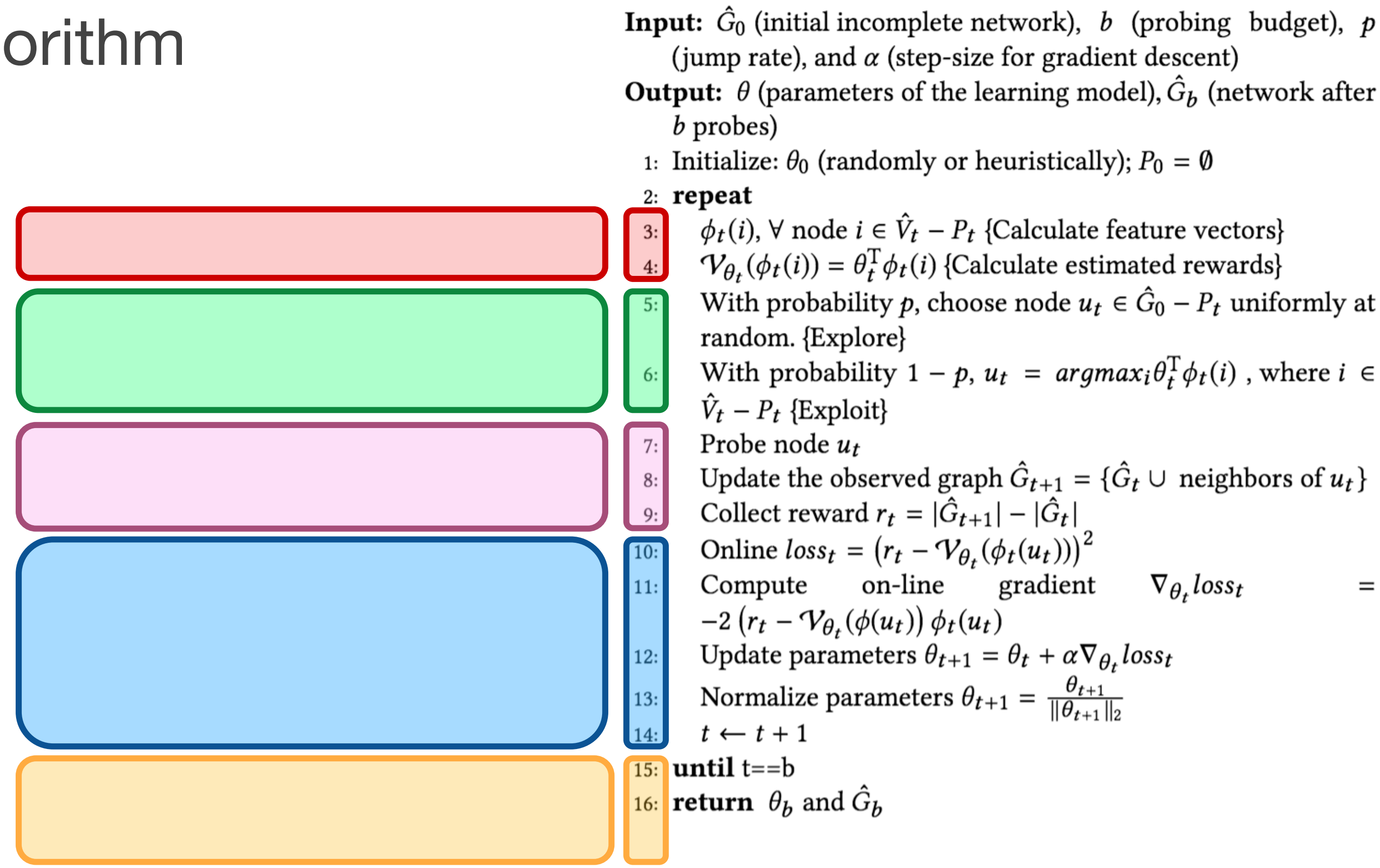
NOL algorithm



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NOL algorithm

Observe the current state

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NOL algorithm

Input: $\hat{G}_0$ (initial incomplete network), b (probing budget), p (jump rate), and α (step-size for gradient descent)

Output: θ (parameters of the learning model), $\hat{G}_b$ (network after b probes)

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Observe the current state

Regression weights

Feature vector of node i

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NOL algorithm

Observe the current state

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NOL algorithm

Observe the current state

Choose the next action

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Observe the current state

Choose the next action

Take action, update network, collect reward

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NOL algorithm

Observe the current state

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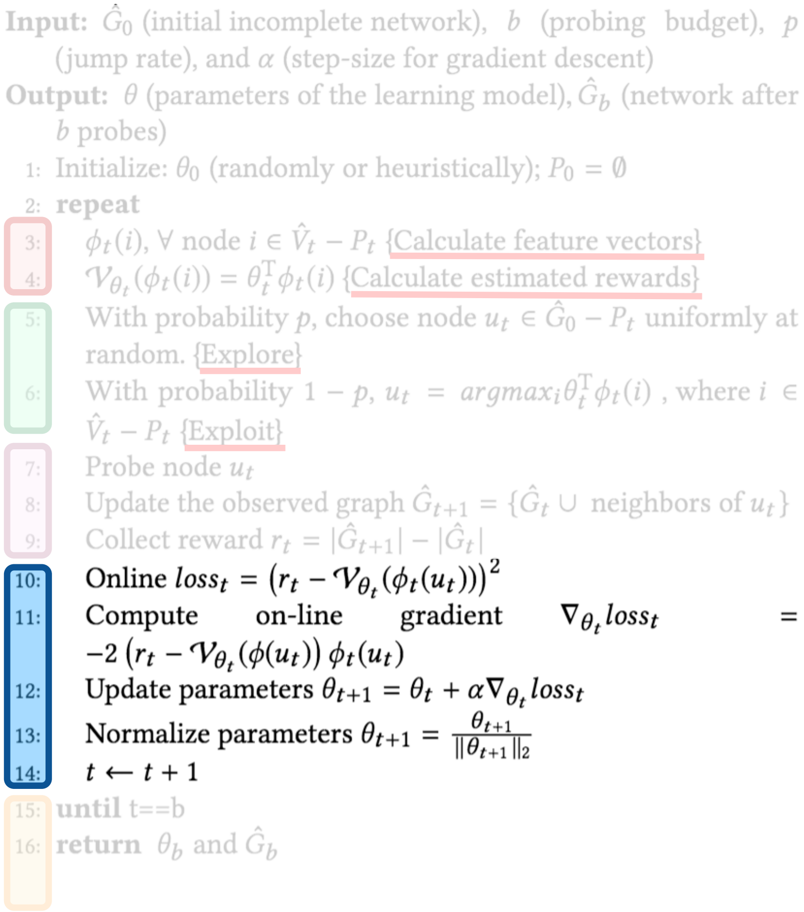

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NOL algorithm

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Online linear regression following Strehl et al., NIPS 2008.

NOL algorithm

Observe the current state

Choose the next action

Take action, update network, collect reward

Update parameters

Repeat until budget depleted

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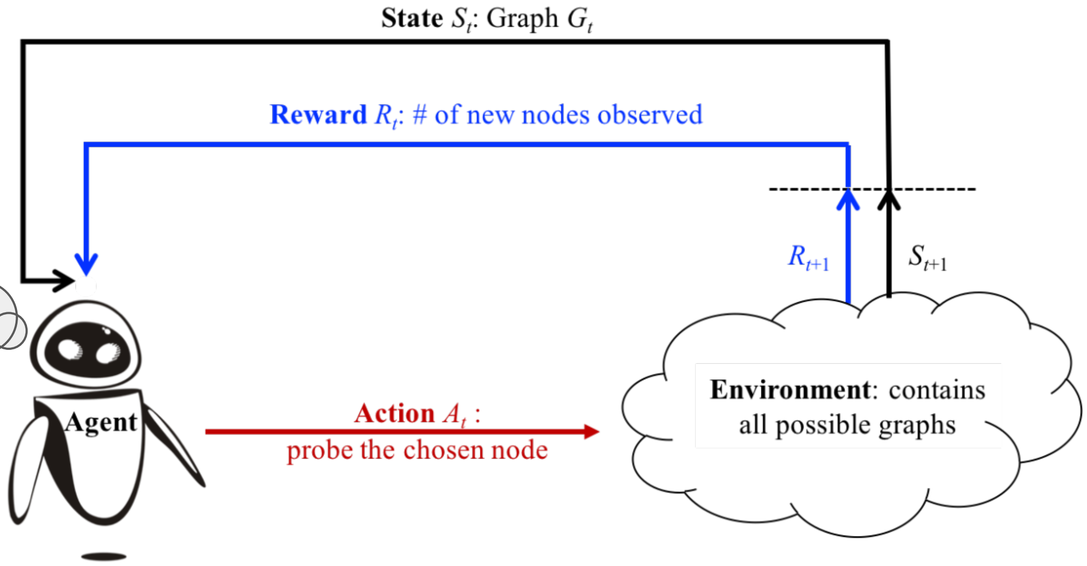
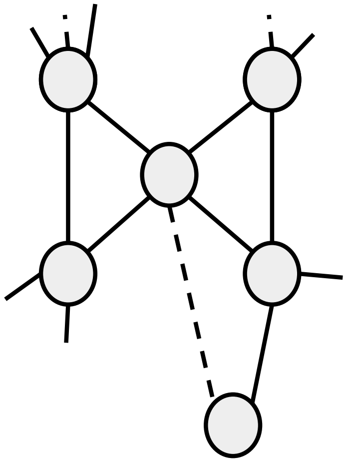
NOL algorithm

1. Observe the current state
2. Choose the next action
3. Take action, update network, collect reward
4. Update parameters
5. Repeat until budget depleted



Features



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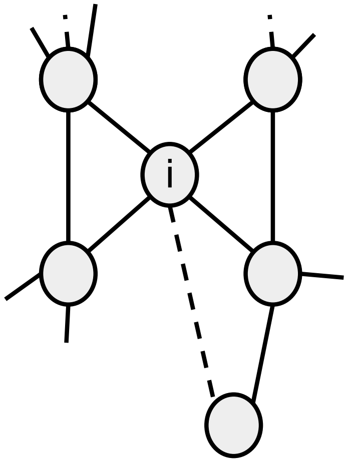
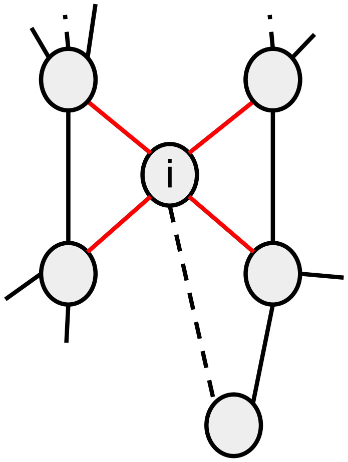


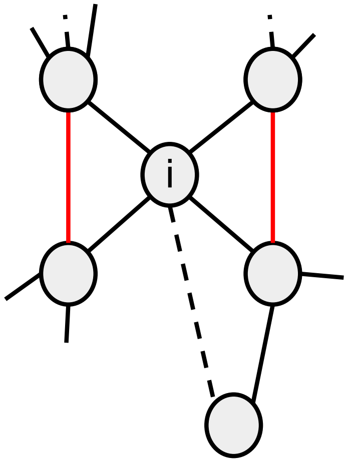
Feature: In-sample degree

$$\phi_{\mathbf{i,0}} = \hat{d}_i$$



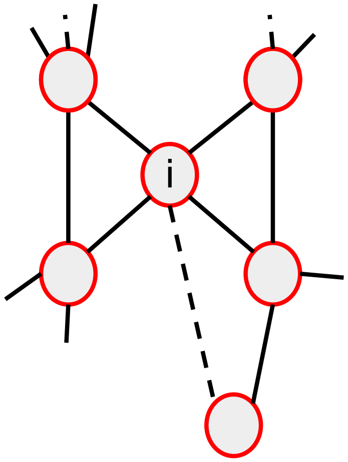
Feature: In-sample clustering coefficient

$$\phi_{\mathbf{i,1}} = \hat{cc}_i$$



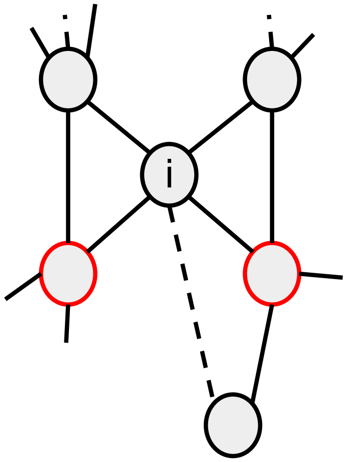
Feature: Normalized size of connected component

$$\phi_{\mathbf{i,2}} = CompSize_i$$



Feature: Fraction of probed neighbors

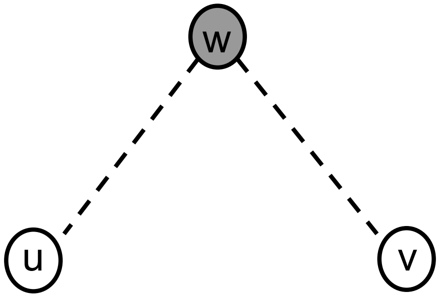
$$\phi_{\mathbf{i,3}} = ProbedNeighbors_i$$



Feature: Lost Reward

$$\phi_{\mathbf{i,4}} = LostReward_i$$

Key idea: The order in which we probe nodes can impact the reward they yield.



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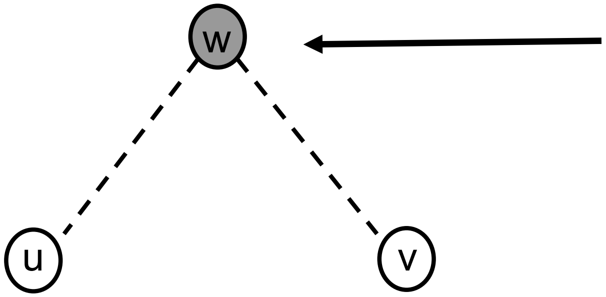

Unobserved node (potential reward for u **or** v)

Partially observed nodes

Research Question: Given a networked dataset, and limited resources to collect more data, **how can you get the most bang for your buck?**

- Potential bang for your buck **depends on network structure!**

Heterogeneity in network properties creates a learning “spectrum”

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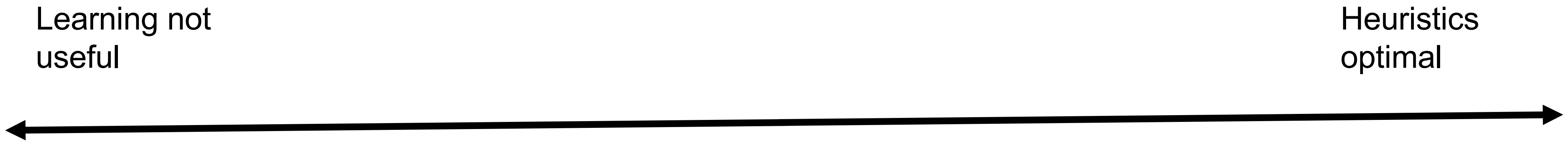
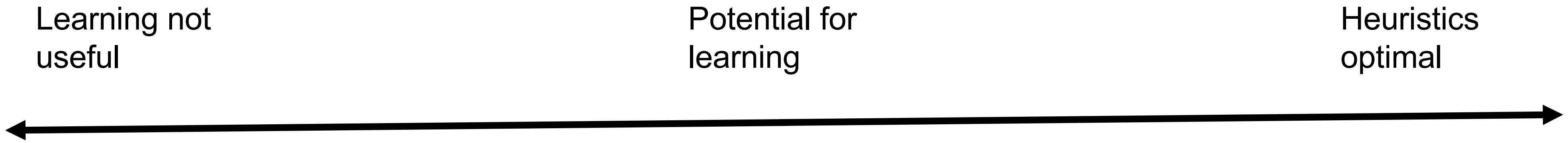


Learning not
useful

Heuristics
optimal

Heterogeneity in network properties creates a learning “spectrum”

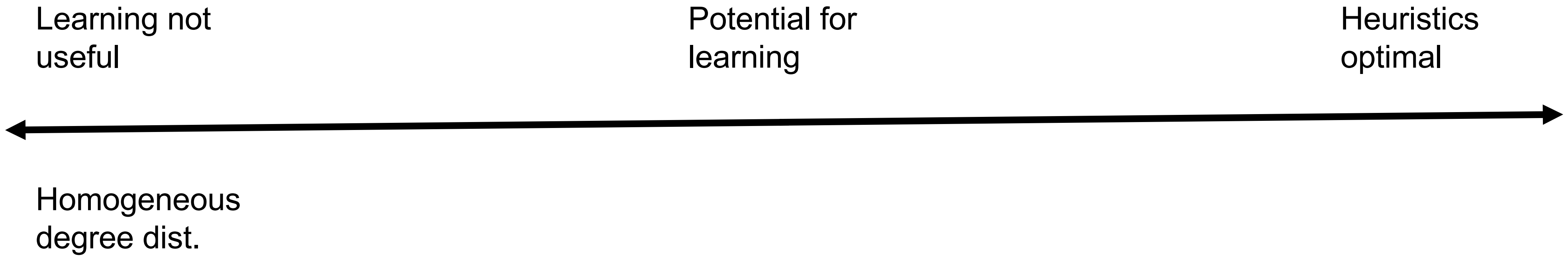


Learning not useful

Potential for learning

Heuristics optimal

Heterogeneity in network properties creates a learning “spectrum”



Learning not
useful

Potential for
learning

Heuristics
optimal

Homogeneous
degree dist.

Heterogeneity in network properties creates a learning “spectrum”

Learning not useful

Potential for learning

Heuristics optimal[1]

Homogeneous degree dist.

Heterogenous degree dist.

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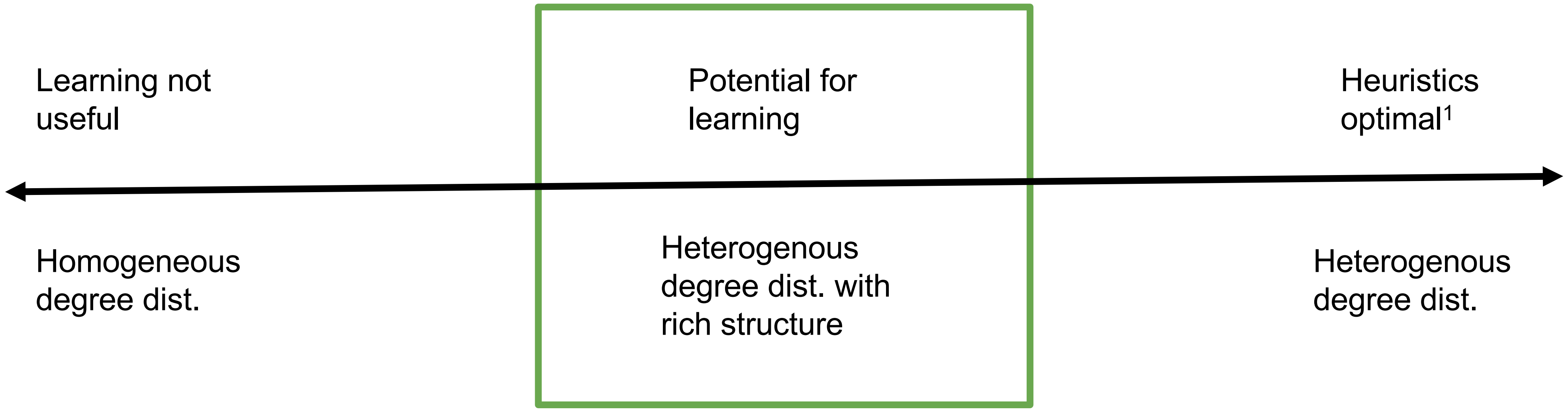
[1] Avrachenkov et al. *INFOCOM WKSHPS* (2014)

Heterogeneity in network properties creates a learning “spectrum”



Learning not useful

Potential for learning

Heuristics optimal[1]

Homogeneous degree dist.

Heterogenous degree dist. with rich structure

Heterogenous degree dist.

[1] Avrachenkov et al. *INFOCOM WKSHPS* (2014)

Experiments

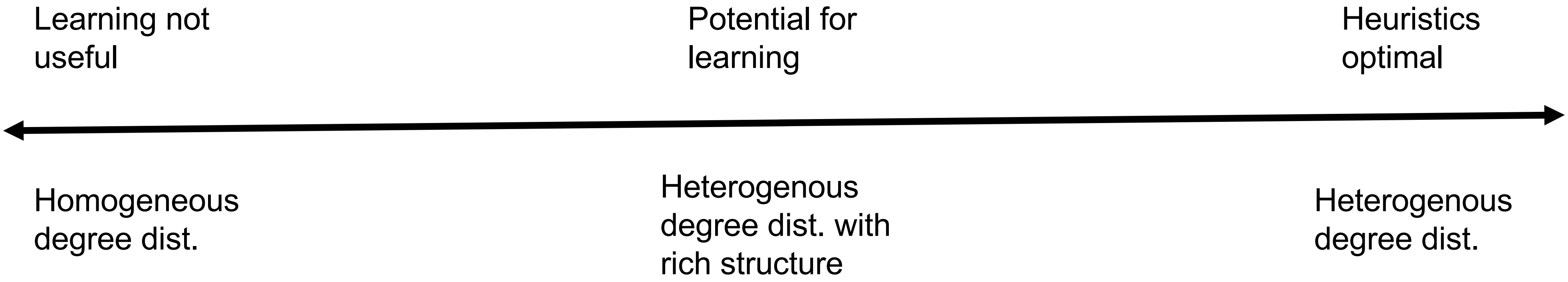
Heuristic Baselines

- High degree
  - Probe the unprobed node with maximum degree
- High degree w/ jump
  - Probe the unprobed node with maximum degree, randomly jump with probability *p*
- Low degree
  - Probe the unprobed node with minimum degree
- Random
  - Probe a node chosen uniformly at random from the unprobed nodes

iKNN-UCB [Madhawa+ ArXiv preprint, 2018]

- K-Nearest Neighbors Upper Confidence Bound
  - Nonparametric multi-armed bandit approach
- Choose node to probe by combining nearest neighbor reward information (based on Euclidean distance between feature vectors) + extent of previous exploration of similar actions

Heterogeneity in network properties creates a learning “spectrum”



Learning not useful

Homogeneous degree dist.

Potential for learning

Heterogenous degree dist. with rich structure

Heuristics optimal

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Heterogeneity in network properties creates a learning “spectrum”

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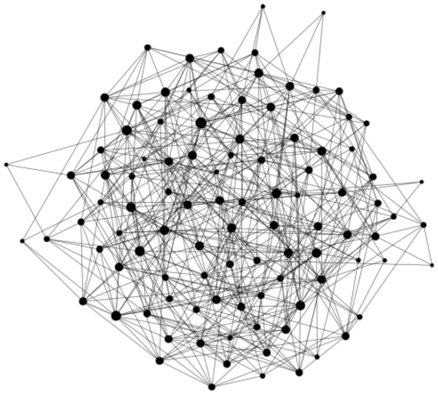
Erdos-Renyi Model

Heterogenous degree dist. with rich structure

Heterogenous degree dist.



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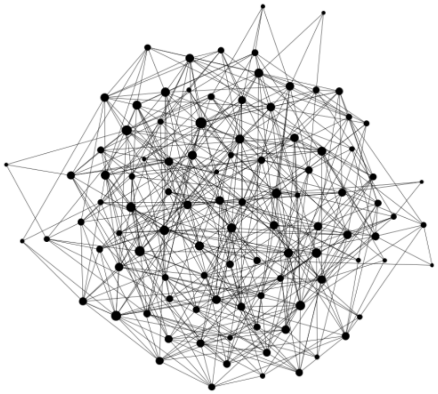
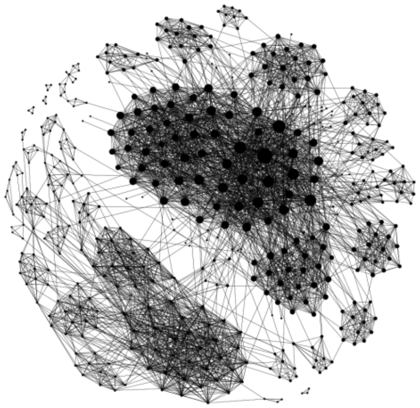
Learning not useful

Erdos-Renyi Model



Potential for learning

BTER Model[1]



Heuristics Optimal

Heterogenous degree dist.

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[1] C. Seshadhri et al. *Physical Review E* (2012)

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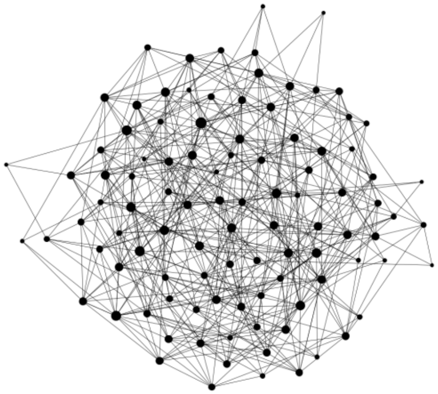
Learning not useful

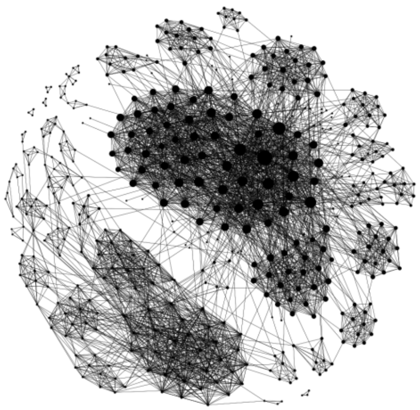
Potential for learning

Heuristics Optimal

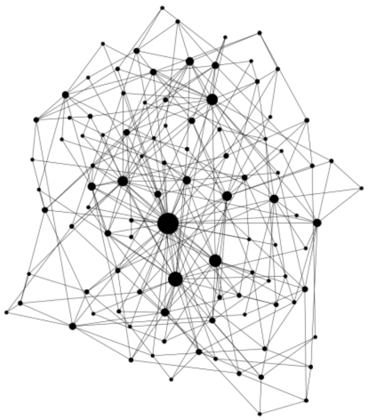
Erdos-Renyi Model



BTER Model[1]



Barabasi-Albert Model



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Results - Learning Spectrum

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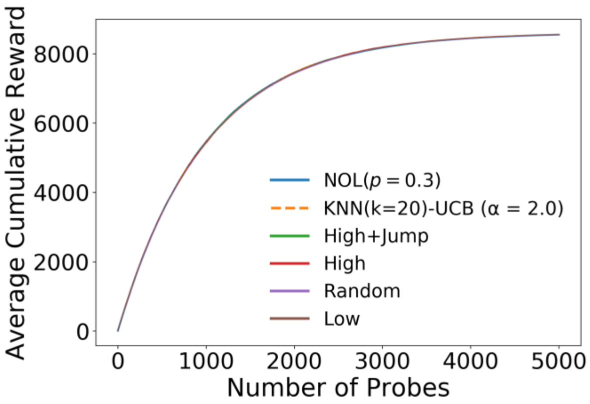
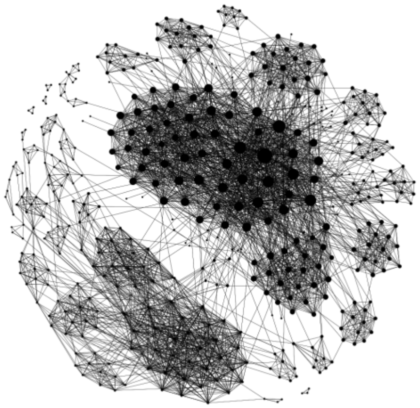
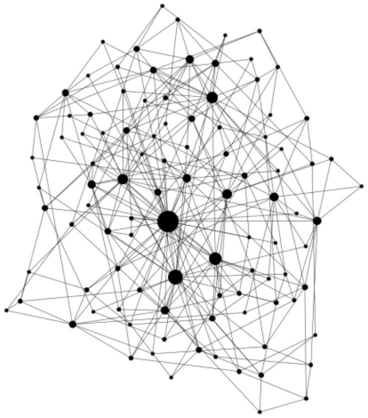


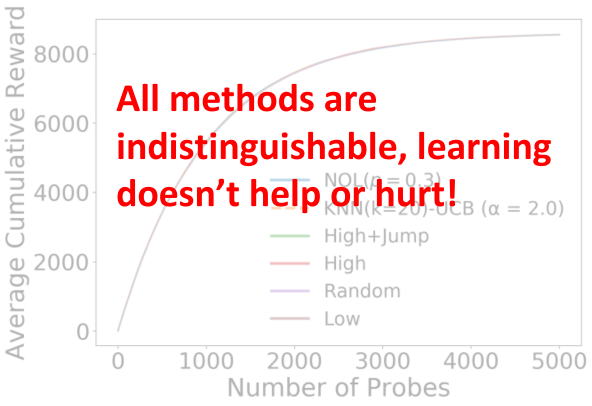




Results - Learning Spectrum

Learning not useful

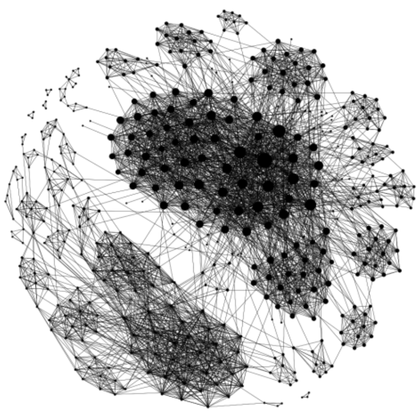
Erdos-Renyi Model



All methods are indistinguishable, learning doesn't help or hurt!

Potential for learning

BTER Model



Heuristics Optimal

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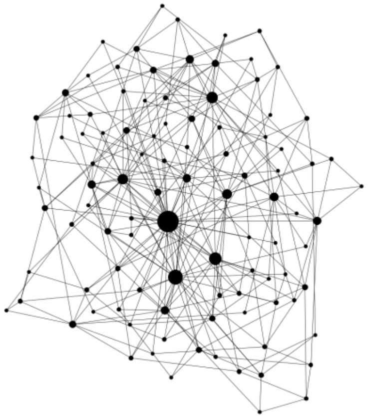

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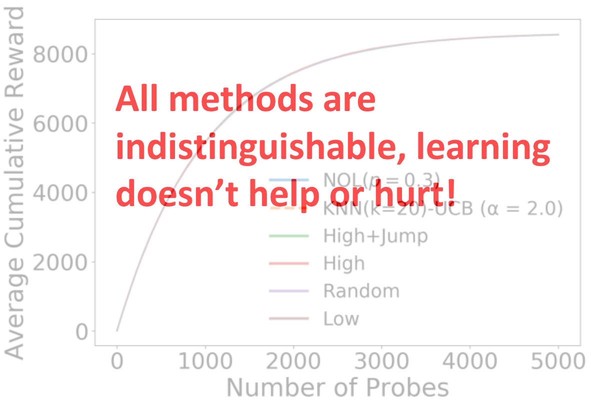

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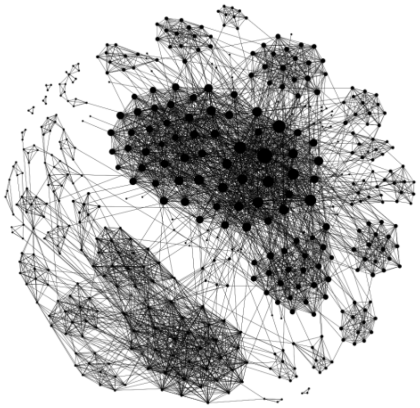

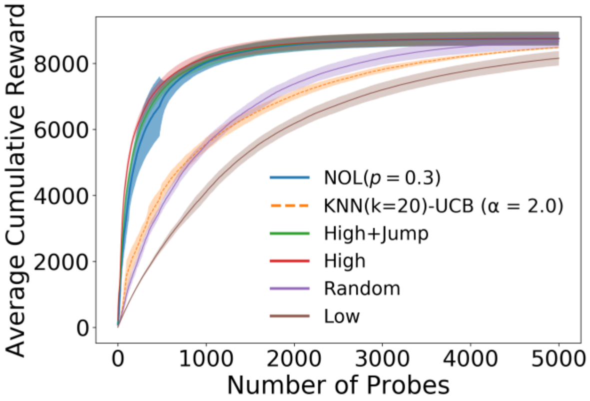

Results - Learning Spectrum

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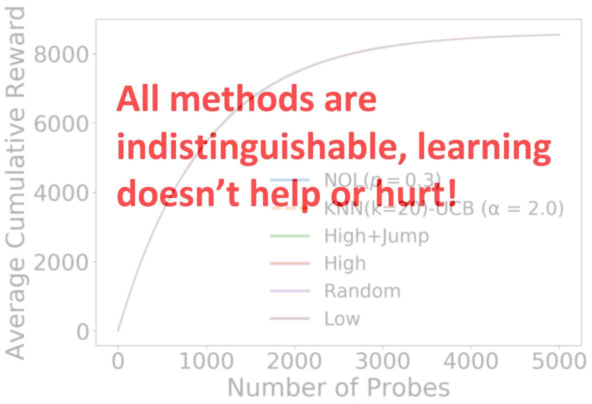

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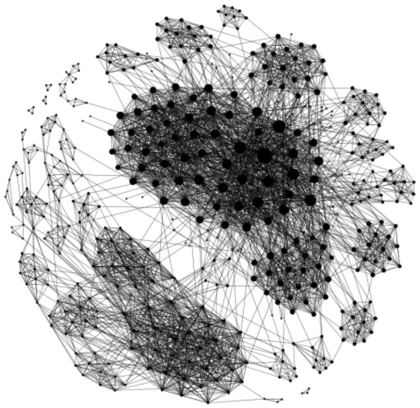

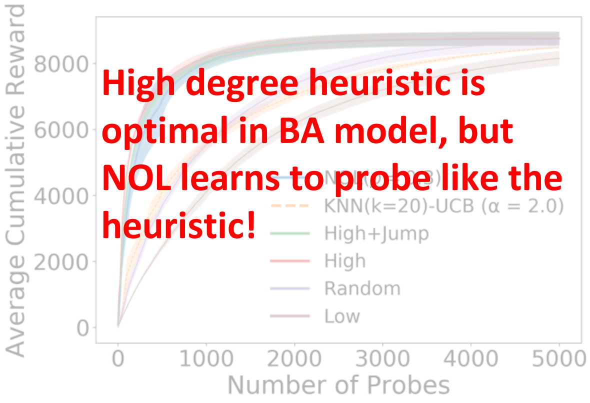

High degree heuristic is optimal in BA model, but NOL learns to probe like the heuristic!

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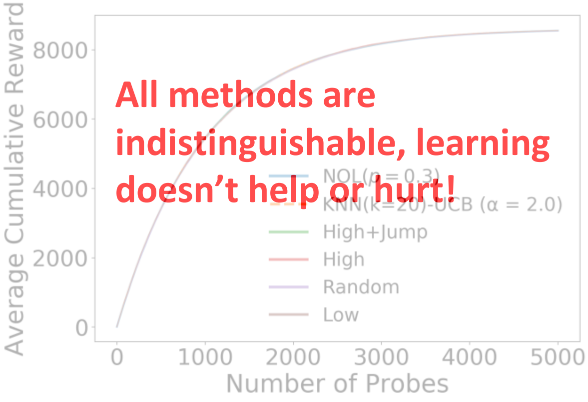

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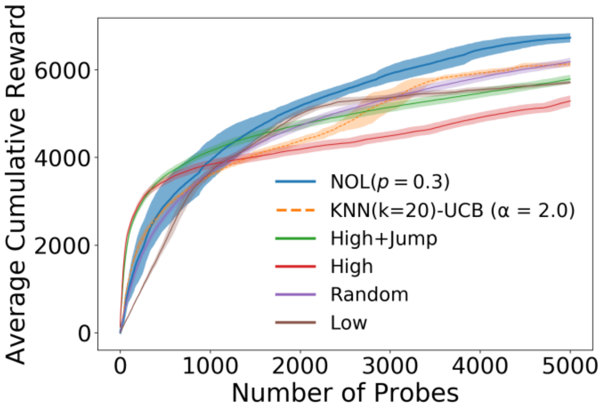
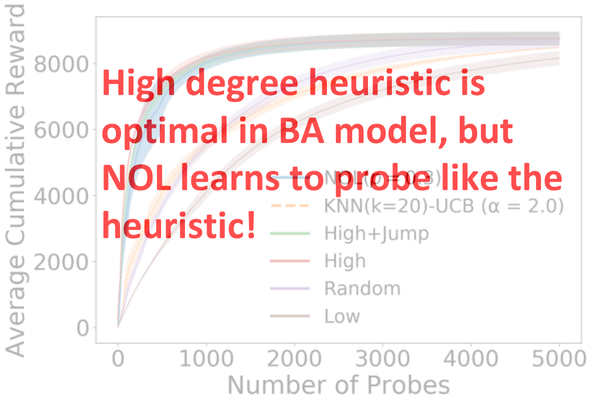




High degree heuristic is optimal in BA model, but NOL learns to probe like the heuristic!

Heterogeneity in network properties creates a learning “spectrum”

Learning Not Useful

Potential for learning

Heuristics Optimal

DBLP

Coauthorship network

N = 6.7k
E = 17k
△s = 21.6k

Cora

Citation network

N = 23k
E = 89k
△s = 78.7k

Enron

Email Communication Network

N = 36.7k
E = 184k
△s = 727k

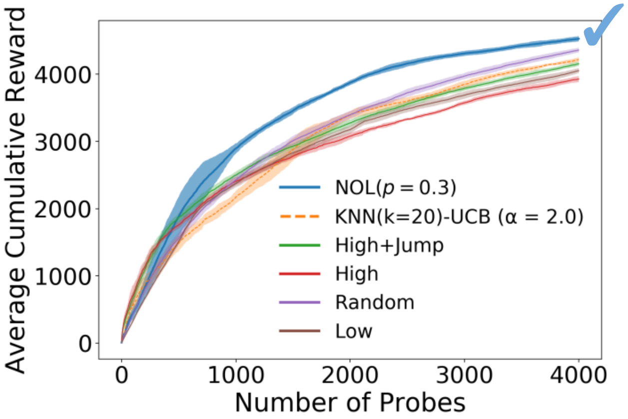
Heterogeneity in network properties creates a learning “spectrum”

Learning Not
Useful

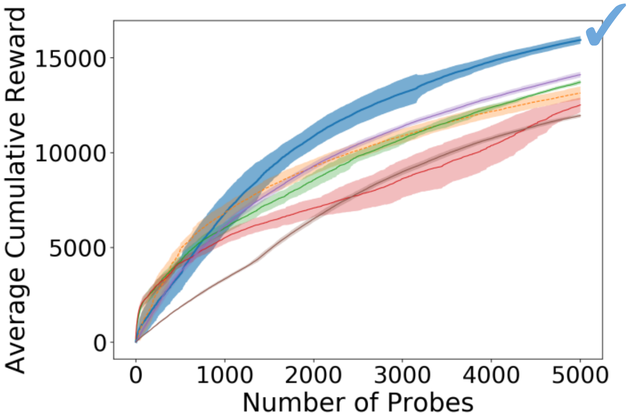
Potential for
learning

Heuristics
Optimal

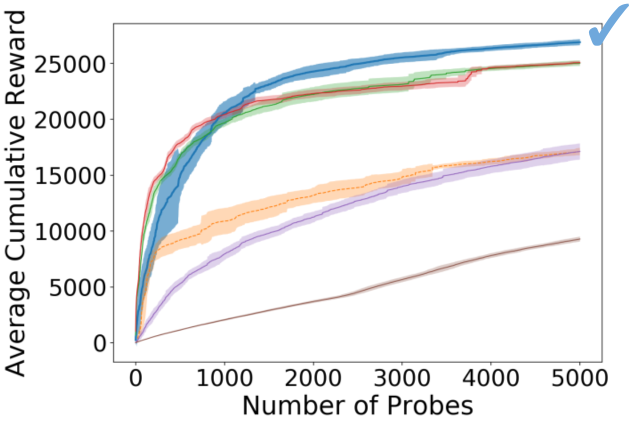
DBLP



Cora



Enron



Summary

- **Network Online Learning** can learn to probe online with minimal assumptions
- Success is tied to properties of the underlying network:
  - Spectrum based on objective function being maximized (degree distribution in these experiments)
  - NOL can learn to behave like the optimal heuristic
- Preliminary experiments suggest some real world complex networks fall in the “learnable” category

Thanks!

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Slides at http://eliassi.org/larock_netsci18.pdf